Solving Equations Involving Percent

Is-Of Statements:

All percent problems can be written in the same basic form which I will call an Is-Of statement.

ex. 25% of 40 is 10

An Is-Of statement involves three numbers; the percent, the" Is" number and the "Of" number.

1. The percent contains the percent sign.
2. The "Of" number always follows the word "of".
3. The "Is" number will be the number preceding or following the word "is".

Example: Given the Is-Of statement

10% of 20 is 2,

identify the percent, the "is" number and the "of" number.

Solution: 10% is the percent, 20 is the "of" number (follows "of"), and 2 is the "is" number (follows "is").

Example: Given the Is-Of statement

20% of 40 is 8,

identify the percent, the "is" number and the "of" number.

Solution: 20% is the percent, 40 is the "of" number (follows "of"), and 8 is the "is" number (follows "is").

Example: Given the Is-Of statement

120 is 200% of What Number,

identify the percent, the "is" number and the "of" number.

Solution: 200% is the percent, "What Number" is the "of" number (follows "of"), and 120 is the "is" number (precedes "is").

Example: Given the Is-Of statement

X is N% of Y,

identify the percent, the "is" number and the "of" number.

Solution: N% is the percent, "Y" is the "of" number (follows "of"), and X is the "is" number (precedes "is").

Translating Is-Of Statements to Mathematical Symbols:

Mathematically the word “of” implies multiplication and the word “is” implies equality. Consequently we can translate an Is-Of statement into a mathematical statement by making these substitutions. In addition, a percent must always be converted into a decimal number by moving the decimal point two places to the left. Finally use a variable (N) to represent the unknown quantity.

Example: 20% of 80 is what number?

Solution:

20% of 80 is N

$$0.20(80) = N$$
$$16 = N$$

Example: What number is 40% of 90?

Solution:

N is 40% of 90

$$N = 0.4(90)$$
$$N = 36$$

Example: 90 is 30% of what number?

Solution:

90 is 30% of N

$$90 = 0.3N$$
$$N = \frac{90}{0.3}$$
$$N = 300$$

Example: 12% of what number is 40?

Solution:

12% of N is 40

$$0.12N = 40$$
$$N = 333.34$$

Example: 9 is what percent of 72?

Solution:

9 is N% of 72

$$9 = N\%(72)$$
$$\frac{9}{72} = N\%$$
$$0.125 = N\%$$
$$N = 12.5\%$$

Solving Percent Problems:

When solving a problem involving percent, use the following procedure.

1. Write the problem as an Is-Of statement.
2. Translate the Is-Of statement to mathematical symbols. That is, change the word “of” to multiplication, the word “is” to and equal sign change the percent to a decimal number and represent the unknown number with a variable.
3. Solve for the variable.

Example: Last year Jordan paid 35% of his income in taxes. How much did he pay in taxes if he earned $80,000?

Solution:

What is 35% of $80,000?

$$N = 0.35(80{,}000)$$
$$N = 28{,}000$$

Example: Only 5% of the students in a particular math class can earn an A If 3 students earned an A, how many students are in the class?

Solution:

3 is 5% of what number?

$$3 = 0.05N$$
$$N = \frac{3}{0.05}$$
$$N = 60$$

Example: A company owns a fleet of 15 aircraft. 2 aircraft are down for maintenance. What percent of the fleet is down for maintenance?

Solution:

2 is N% of 15

$$2 = N\%(15)$$
$$N\% = \frac{2}{15}$$
$$N\% = 0.1334$$
$$N = 13.34\%$$

Percent Increase or Decrease Problems

These problems deal with quantities that are changing; that is, increasing or decreasing. If a quantity increases we have a percent increase problem and if a quantity decreases we have a percent decrease problem.

To solve a percent increase or decrease problem:

1. Find the amount of increase or decrease.
2. Divide this amount by the original number.
3. Multiply by 100.

Example: A quantity increases from 50 units to 65 units. What is the percent increase?

Solution: The amount of increase is 65 – 50 = 15. Then,

$$N\% = \frac{15}{50}(100) = 0.3(100)$$
$$N = 30\%$$

Note that this example could have also been done as an IS-OF statement as well. Once the amount of increase is determined we can write:

$$\text{15 is N\% of 50}$$
$$15 = N\%(50)$$
$$N\% = \frac{15}{50}(100)$$
$$N = 30\%$$

Example: Jacob’s hourly salary increased from \$12.42 to \$13.85 per hour. What is Jacob’s percent increase in pay?

Solution: The increase is 13.85 – 12.42 = 1.43. Then,

$$N\% = \frac{1.43}{12.42}(100) = 0.1151$$
$$N = 11.51\%$$

Example: Robert’s math average decreased from an 86% to a 75%. What is the percent decrease in his average?

Solution: The amount of decrease is 86 – 75 = 11. Then,

$$N\% = \frac{11}{86}(100) = (0.1279)(100)$$
$$N\% = 12.79\%$$

Applications:

Example: Last year at Grand Canyon University, 70% of the awarded degrees were to undergraduates. If 2,450 degrees were awarded last year, how many were to undergraduates?

Solution: This problem may be restated as an IS-OF statement, then converted into mathematical symbols and solved accordingly.

70% of 2,450 is what number?

$$0.7(2450) = N$$
$$N = 1715$$

Example: If a bank account pays $120.00 in simple interest on an initial balance of $2,000, what is the annual interest rate?

Solution: This problem may be restated as an IS-OF statement, then converted into mathematical symbols and solved accordingly.

120 is N% of 2,000

$$120 = N\%(2000)$$
$$N\% = \frac{120}{2000}$$
$$N\% = 0.06$$
$$N = 6\%$$

Example: Gwen sold her car on consignment. The saleswoman's commission was 10% of the selling price and Gwen received $6,570. Find the selling price of the car.

Solution: This problem may be restated as an IS-OF statement, then converted into mathematical symbols and solved accordingly.

If the commission is 10% of selling price then Gwen will receive 90% of the selling price. Let the selling price equal N.

6570 is 90% of N

$$6570 = 0.90N$$
$$N = \frac{6570}{0.9}$$
$$N = 7{,}300.$$

Therefore the selling price of the car is $7,300

Example: A cattle rancher is going to sell one of his prize bulls at an auction and would like to make \$45,500.00 after paying a 9% commission to the auctioneer. For what selling price will the rancher make this amount of money?

Solution:
Step 1: Analyze the Problem:
If the commission is 9% that means that the rancher takes home 91%. Let x = the selling price. Then the IS-OF statement is

\$45,500 is 91% of what number (selling price.)

Step 2: Create an Equation:
The equation is based on the IS-OF statement.
$45{,}500 = 0.91x$

Step 3: Solve the Equation:
$45{,}500 = 0.91x$

$x = \$50{,}000$

Step 4: State the Conclusion:
The selling price must be \$50,000 in order for the rancher to take home \$45,500.

Example: Between the years 2000 and 2006, the average cost for auto insurance nationwide grew 27%, to \$867.00. What was the average cost in 2000?

Solution:
Step 1: Analyze the Problem:
This is a percent increase problem. The formula for percent increase is always the amount of increase divided by the original number times 100.

Let x = the average cost in 2000. Then we can create the following equation based on percent increase.

Step 2: Create an Equation:
$$\left[\frac{867-x}{x}\right](100) = 27\%$$

Step 3: Solve the Equation:
This is a rational equation. To solve, multiply both sides of the equation by the LCD, which is x.

$$x\left[\frac{867-x}{x}\right] = [0.27]x$$

$$867 - x = 0.27x$$

$$867 = 1.27x$$

$$x = 682.67$$

Step 4: State the Conclusion:
The average cost of auto insurance in 2000 was \$682.67.

Example: What was the MSRP for a Hummer H1 that sold for \$107,272 after an 8% discount?

Solution:
Step 1: Analyze the Problem:
This is a percent problem that may be put into the form of an
IS-OF Statement. If the discount is 8% that means that the selling price is 92% of the MSRP.

Let x = MSRP. Then,

\$107,272 is 92% of what number (MSRP)

Step 2: Create an Equation:

$$107{,}272 = 0.92x$$

Step 3: Solve the Equation:

$$0.92x = 107{,}272$$
$$x = \frac{107{,}272}{0.92}$$
$$x = \$116{,}600$$

Step 4: State the Conclusion:
The MSRP is \$116,600.

Example: A professor wants to supplement his pension with investment interest. If he invests \$28,000 at 6% interest how much does he have to invest at 7% interest to achieve a yield of \$3,500 per year in investment interest.

Solution:
Step 1: Analyze the Problem:
An investment problem is a type of mixture problem. Therefore, it is often advantageous to make a table to organize the information.

Investment	Interest Rate	Amount	Yield
1	6%	28,000	0.06(28,000)
2	7%	x	0.07x

Step 2: Create an Equation:
The equation is based on the fact that the sum of the yields from the two investments must be equal to his desired yield of \$3,500.

$$0.06(28{,}000) + 0.07x = 3500$$

Step 3: Solve the Equation:

$$0.06(28{,}000) + 0.07x = 3500$$
$$1680 + 0.07x = 3500$$
$$0.07x = 1820$$
$$x = 26{,}000$$

Step 4: State the Conclusion:
\$26,000 must be invested at 7%.

Example: A salesperson used his $3,500 year-end bonus to purchase some old coins, with hopes of earning 15% annual interest on the gold coins and 12% annual interest on the silver coins. If he saw a return on his investment of $480.00 the first year, how much did he invest in each type of coin?

Solution:
Step 1: Analyze the Problem:
An investment problem is a type of mixture problem. Therefore, it is often advantageous to make a table to organize the information.

Coins	Earning Rate	Amount	Return
Gold	15%	x	0.15x
Silver	12%	3500-x	0.12(3500-x)

Step 2: Create an Equation:
The equation is based on the fact that the sum of the yields from the two investments must be equal to his total return of $480.00

$$0.15x + 0.12(3500 - x) = 480$$

Step 3: Solve the Equation:

$$0.15x + 0.12(3500 - x) = 480$$
$$0.15x + 420 - 0.12x = 480$$
$$0.03x = 60$$
$$x = \$2{,}000$$

Step 4: State the Conclusion:
$2,000 must be invested in gold and $1500 in silver.

Example: Equal amounts are invested in each of three accounts paying 7%, 8%, and 10.5% interest annually. If one year’s combined interest income is $1,249.50, how much is invested in each account?

Solution:
Step 1: Analyze the Problem:
An investment problem is a type of mixture problem. Therefore, it is often advantageous to make a table to organize the information. We can let x = the amount of each investment because we are told the amounts are equal.

Investment	Interest Rate	Amount	Income
1	7%	x	0.07x
2	8%	x	0.08x
3	10.5%	x	0.105x

Step 2: Create an Equation:
The equation is based on the sum of the yields from the three investments being equal to the total income of $1,249.5.

$$0.07x + 0.08x + 0.105x = 1249.5$$

Step 3: Solve the Equation:

$$0.07x + 0.08x + 0.105x = 1249.5$$
$$0.255x = 1249.5$$
$$x = \$4{,}900$$

Step 4: State the Conclusion:

$4,900 must be invested in each account.