2.2-Frequency Distributions

When working with large data sets, it is often helpful to organize and summarize data by constructing a table called a frequency distribution, defined later. Because computer software and calculators can generate frequency distributions, the details of constructing them are not as important as what they tell us about data sets. It helps us understand the nature of the *distribution* of a data set.

Frequency Distribution:

Also called a Frequency Table shows how a data set is partitioned among all of several categories (or classes) by listing all of the categories along with the number of data values in each of the categories.

Example: Given Table 2-1 below which gives the pulse rates of females and males, construct a frequency table for the pulse rates of the females.

Table 2-1 Pulse Rates (beats per minute) of Females and Males

Females																			
76	72	88	60	72	68	80	64	68	68	80	76	68	72	96	72	68	72	64	80
64	80	76	76	76	80	104	88	60	76	72	72	88	80	60	72	88	88	124	64
Males																			
68	64	88	72	64	72	60	88	76	60	96	72	56	64	60	64	84	76	84	88
72	56	68	64	60	68	60	60	56	84	72	84	88	56	64	56	56	60	64	72

Solution: The ***frequency*** for a particular class is the number of original values that fall into that class. The classes are the intervals in the left-hand column.

Table 2-2 Pulse Rates of Females

Pulse Rate	Frequency
60–69	12
70–79	14
80–89	11
90–99	1
100–109	1
110–119	0
120–129	1

Reasons for Constructing Frequency Distributions:

1. Large sets of data can easily be summarized.
2. The nature of the data can be analyzed.
3. We have the basis for constructing important graphs.

Lower Class Limits:

The smallest numbers that can actually belong to different classes.

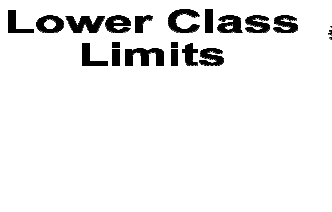

Table 2-2 Pulse Rates of Females

Pulse Rate	Frequency
60–69	12
70–79	14
80–89	11
90–99	1
100–109	1
110–119	0
120–129	1

Upper Class Limits:

The largest numbers that can actually belong to different classes

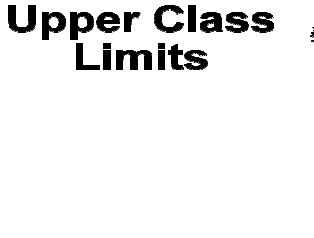

Table 2-2 Pulse Rates of Females

Pulse Rate	Frequency
60–69	12
70–79	14
80–89	11
90–99	1
100–109	1
110–119	0
120–129	1

Class Boundaries:

The numbers used to separate classes, but without the gaps created by class limits

Table 2-2 Pulse Rates of Females

Pulse Rate	Frequency
60–69	12
70–79	14
80–89	11
90–99	1
100–109	1
110–119	0
120–129	1

Class Midpoints:

The values in the middle of the classes and can be found by adding the lower class limit to the upper class limit and dividing the sum by two.

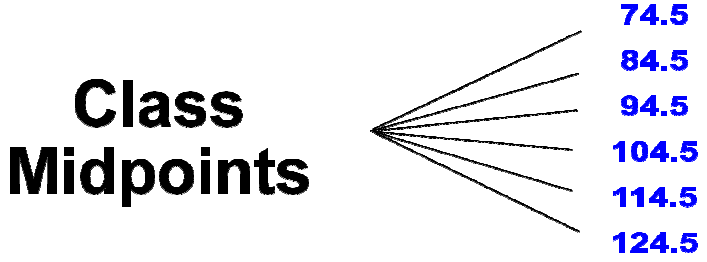

Table 2-2 Pulse Rates of Females

Pulse Rate	Frequency
60–69	12
70–79	14
80–89	11
90–99	1
100–109	1
110–119	0
120–129	1

Class Width:

The difference between two consecutive lower class limits or two consecutive lower class boundaries.

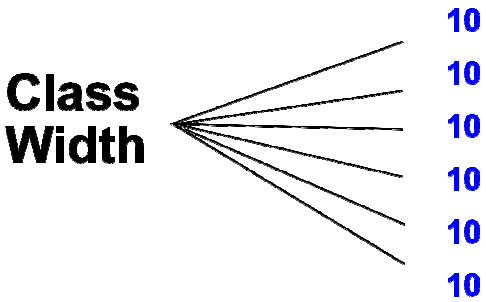

Table 2-2 Pulse Rates of Females

Pulse Rate	Frequency
60–69	12
70–79	14
80–89	11
90–99	1
100–109	1
110–119	0
120–129	1

Example: The following frequency distribution analyzes the scores on a math test. Determine the lower and upper class limits, class width, midpoints and boundaries.

Scores	Number of students
40–59	2
60–75	4
76–82	6
83–94	15
95–99	5

Solution:

Lower Limits – 40, 60, 76, 83, 95
Upper Class Limits – 59, 75, 82, 94, 99
Class Width – 10
Class Midpoints – 49.5, 67.5, 79, 88.5, 97
Class Boundaries – 59.5, 75.5, 82.5, 94.5

Normal Frequency Distribution:

A normal distribution is somewhat "bell" shaped, that is it starts low then increases to one or two high frequencies, then decreases to a low frequency. This distribution is approximately symmetric.

Example: Using a strict interpretation of the relevant criteria characterizing a normal distribution, does the frequency distribution below appear to have a normal distribution?

Closing Share Price	Frequency
0–5	2
6–10	10
11–15	13
16–20	7

Solution: because this distribution starts low then increases to one or two high frequencies, then decreases to a low frequency it is normally distributed.

Example: Using a strict interpretation of the relevant criteria characterizing a normal distribution, does the frequency distribution below appear to have a normal distribution?

Closing Share Price	Frequency
0–5	2
6–10	5
11–15	15
16–20	27

Solution: Because this distribution starts low then continues to increase, it is not normally distributed.

Constructing a Frequency Table:

1. Determine the number of classes.
2. Calculate the class width using the following formula:

$$\frac{Maximum Value - Minimum Value}{Number.of.Classes}$$

3. Choose the minimum data value or a convenient value below it as a starting point.
4. Using the first lower class limit and class width, proceed to list the other lower class limits.
5. List the lower class limits in a vertical column and proceed to enter the upper class limits.
6. Take each individual data value and put a tally mark in the appropriate class. Add the tally marks to get the frequency.

Example: Lori asked 24 students how many hours they had spent doing homework during the previous week. The results are shown below.

10	11	10	8	10	10	14	13	10	9	13	11
11	13	10	11	13	10	11	13	11	13	13	8

Construct a frequency distribution. Use 4 classes, a class width of 2 hours, and a lower limit of 8

Solution:

Hours	Frequency
8-9	3
10-11	13
12-13	7
14-15	1

Example: On a math test, the scores of 24 students were:

99	78	79	67	79	79	99	89	79	68	88	78
78	88	79	78	88	79	78	89	78	88	89	67

Construct a frequency distribution. Use 4 classes beginning with a lower class limit of 60.

Solution:

Grades	Frequency
60 - 69	3
70 - 79	12
80-89	7
90-99	2

Relative Frequency Distributions:

A relative frequency distribution includes the same class limits as a frequency distribution, but the frequency of a class is replaced with a relative frequencies (a proportion) or a percentage frequency (a percent)

$$\text{relative frequency} = \frac{\text{class frequency}}{\text{sum of all frequencies}}$$

$$\text{percentage frequency} = \frac{\text{class frequency}}{\text{sum of all frequencies}} \times 100\%$$

Example: The frequency distribution for the weekly incomes of students with part-time jobs is given below. Construct the corresponding relative frequency distribution. Round relative frequencies to the nearest hundredth of a percent if necessary.

Income ($)	Frequency
200–300	60
301–400	73
401–500	91
501–600	89
More than 600	15

Solution: There is a total frequency of 328. To find the relative frequency for each class, divide the frequency of that class by 328 then multiply by 100 to make it a percent frequency.

Income ($)	Relative Frequency
200–300	18.29%
301–400	22.26%
401–500	27.74%
501–600	27.13%
More than 600	4.57%

Cumulative Frequency Distributions:

A cumulative frequency table adds each class frequency to the next.

Table 2-2 Pulse Rates of Females

Pulse Rate	Frequency
60-69	12
70-79	14
80-89	11
90-99	1
100-109	1
110-119	0
120-129	1

Table 2-4 Cumulative Frequency Distribution of Pulse Rates of Females

Pulse Rate	Cumulative Frequency
Less than 70	12
Less than 80	26
Less than 90	37
Less than 100	38
Less than 110	39
Less than 120	39
Less than 130	40

Cumulative Frequencies

Example: Construct the cumulative frequency distribution that corresponds to the given frequency distribution.

Weight (oz)	Number of Stones
1.2–1.6	5
1.7–2.1	2
2.2–2.6	5
2.7–3.1	5
3.2–3.6	13

Solution:

Weight (oz)	Cumulative Frequency
Less than 1.7	5
Less than 2.2	7
Less than 2.7	12
Less than 3.2	17
Less than 3.7	30

Summary

In this Section we have discussed

- **Important characteristics of data**
- **Frequency distributions**
- **Procedures for constructing frequency distributions**
- **Relative frequency distributions**
- **Cumulative frequency distributions**