Complements: The Probability of “At least One”

- The term “at least one” is equivalent to “one or more.”
- The complement of getting at least one item of a particular type is that you get no items of that type.

Example: Identify the complement of the statement “Of ten adults, at least one of them has high blood pressure.”

Solution: None of the adults have high blood pressure.

Example: Identify the complement of the statement “When several textbooks are edited, none of them are found to be free of errors.”

Solution: At least one of the textbooks is free of errors.

Finding Probability of “At least One.”

- To find the probability of at least one of something, calculate the probability of none, and then subtract that result from 1. That is,

\[ P(\text{at least one}) = 1 - P(\text{none}) \]

Example: An unprepared student makes random guesses for the ten true-false questions on a quiz. Find the probability that there is at least one correct answer.

Solution: Let event \( A \) = there is at least one correct answer. Then, the complement \( \bar{A} \) will be that there are no correct answers.

\[
P(\bar{A}) = \left(\frac{1}{2}\right)^{10} = \frac{1}{1024} = 0.001
\]

Then, \( P(A) = 1 - P(\bar{A}) = 1 - 0.001 = 0.999 \)

The probability that there is at least one correct answer is 0.999.

Note that the event \( P(\bar{A}) \) is a compound event consisting of 10 simple independent events.

Example: In a batch of 8,000 clock radios 5% are defective. A sample of 14 clock radios is randomly selected without replacement from the 8,000 and tested. The entire batch will be rejected if at least one of those tested is defective. What is the probability that the entire batch will be rejected?

Solution: Let event \( A \) = there is at least one defective radio selected. Then, the complement \( \bar{A} \) will be that there are no defective radios selected. 95% of the radios are not defective. Because we are selecting a sample of only 14 out of 8,000, we can use the 5% rule even though we are selecting without replacement.

\[
P(\bar{A}) = (0.95)^{14} = 0.488
\]

Then, \( P(A) = 1 - P(\bar{A}) = 1 - 0.488 = 0.512 \)

The probability that at least one radio is defective and the whole batch is rejected is 0.512.
**Example:** In a blood testing procedure, blood samples from 6 people are combined into one mixture. The mixture will only test negative if all the individual samples are negative. If the probability that an individual sample tests positive is 0.11, what is the probability that the mixture will test positive?

**Solution:** Let event $A$ = there is at least one positive blood sample. Then, the complement $\overline{A}$ will be that there are no positive blood samples.

$$P(\overline{A}) = (0.89)^6 = 0.497$$

Then, $P(A) = 1 - P(\overline{A}) = 1 - 0.497 = 0.503$

The probability that at least one blood sample is positive and therefore the mixture is positive is 0.503.

**Example:** One design feature contributing to reliability is the use of redundancy, whereby critical components are duplicated so that if one fails, the other will work. For example, single-engine aircraft now have two independent electrical systems so that if one electrical system fails, the other can continue to work so that the engine does not fail.

If the probability of an electrical system failure is 0.001, determine the probability of not having an electrical failure. If the aircraft has two electrical systems, what is the probability that the aircraft will operate with a working electrical system.

**Solution:** The probability of not having an electrical failure is the complement of having an electrical failure. Therefore, $P($ not having electrical failure $) = 1 - 0.001 = 0.999$.

The probability that both electrical systems will fail is $0.001 \times 0.001 = 0.000001$. The probability of both systems not failing is therefore $1 - 0.000001 = 0.999999$.

The redundancy decreases the probability of an electrical failure from $\frac{1}{1000}$ to $\frac{1}{1000000}$.

**Conditional Probability:**
A conditional probability of an event is a probability obtained with the additional information that some other event has already occurred. $P(B|A)$ denotes the conditional probability of event $B$ occurring, given that event $A$ has already occurred, and it can be found by dividing the probability of events $A$ and $B$ both occurring by the probability of event $A$:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

**Intuitive Approach to Conditional Probability:**
Finding a conditional probability is often much less difficult if done intuitively. The conditional probability of $B$ given $A$ can be found by assuming that event $A$ has occurred, and then calculating the probability that event $B$ will occur.
Example: The table below shows the soft drinks preferences of people in three age groups.

<table>
<thead>
<tr>
<th></th>
<th>cola</th>
<th>root beer</th>
<th>lemon-lime</th>
</tr>
</thead>
<tbody>
<tr>
<td>under 21 years of age</td>
<td>40</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>between 21 and 40</td>
<td>35</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>over 40 years of age</td>
<td>20</td>
<td>30</td>
<td>35</td>
</tr>
</tbody>
</table>

If one of the 255 subjects is randomly selected, find the probability that the person drinks root beer given that they are over 40.

Solution: First let's use the intuitive approach. Let event $A = a$ person is over 40 and event $B = a$ the person drinks root beer. We will find $P(B|A)$ which means we are finding the probability that a person drinks root beer while assuming that they are over 40.

$$P(B|A) = \frac{30}{85} = 0.353$$

Now, using the formula we find the probabilities as follows:

$$P(A) = \frac{85}{255} \quad P(B) = \frac{75}{255} \quad P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{30}{255}}{\frac{85}{255}} = \frac{30}{85} = 0.353$$

Example: The following table contains data from a study of two airlines which fly to Small Town, USA.

<table>
<thead>
<tr>
<th></th>
<th>Number of flights which were on time</th>
<th>Number of flights which were late</th>
</tr>
</thead>
<tbody>
<tr>
<td>Podunk Airlines</td>
<td>33</td>
<td>6</td>
</tr>
<tr>
<td>Upstate Airlines</td>
<td>43</td>
<td>5</td>
</tr>
</tbody>
</table>

If one of the 87 flights is randomly selected, find the probability that the flight selected is an Upstate Airlines flight given that it was late.

Solution: The intuitive approach will be easiest. Let event $A = a$ the flight is late and event $B = a$ the flight is with Upstate Airlines. We will find $P(B|A)$ which means we are finding the probability that a flight is with Upstate Airlines given that it was late.

$$P(B|A) = \frac{5}{11} = 0.455$$
Confusion of the Inverse:
To incorrectly believe that $P(A|B)$ and $P(B|A)$ are the same, or to incorrectly use one value for the other is often called confusion of the inverse.

**Example:** let event $A$ = it is midnight and event $B$ = it is dark outside. Find and compare $P(B|A)$ and $P(A|B)$

**Solution:** The $P(B|A) = 1$ while $P(A|B) = 0$